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RITTER'S "ASIA": RUSSIAN ADDENDA.

Eastern Siberia, including Lake Baikal and the Mountains on its North-Western Shore. Vol. II. By P. P. Semenoff, I. D. Cherskiy, and G. G. von Petz. Pp. 630. (Russian: St. Petersburg, 1895.)

THIS new volume, edited by P. P. Semenoff, from the MSS. of I. D. Cherskiy, and containing 630 pages of text, in lieu of the three paragraphs of Ritter's work, is even more interesting than the preceding volume, which was noticed in these columns a short time ago (*NATURE*, vol. I. p. 471). It covers Lake Baikal and the mountains along its north-western shore, and embodies explorations either entirely new or quite unknown even in Russia itself. Moreover, all that has been said concerning the preceding volume, as regards the masterly treatment of the subject and a strict adherence to Ritter's excellent methods—a combination of a minute description of details with broad generalisations drawn out of them—fully applies to this new instalment of the great work undertaken by the Russian Geographical Society. A third volume, containing Transbaikalia and the Gobi, will soon follow—the invaluable collaboration of M. Obrucheff having been secured for this purpose by the editor.

When we cast a glance upon a good orographical map of Asia (*e.g.* Petermann's, in Stieler's "Hand Atlas," or even in the miniature "Taschen Atlas" of the same publishers), we see that the two great plateaus of West and East Asia are fringed along their north-western borders with a chain of great lakes: the Caspian Sea, Lake Balkhash, Ala-kul and Zaisan, Ulungur, Baikal, and Oron; while a succession of large post-Tertiary lakes, now desiccated, which formerly filled the valleys of the Tian-shan, the Altai, the Sayans, and the Muya ridges, complete this chain of depressions along the outer border of the plateaus. Lake Baikal is one of the lakes of this chain—a small remainder only of the great mass of water which formerly filled up the valley of the Irkut, and the lower parts of the eastern tributaries of the present lake, and discharged its waters, as we now learn from the volume under review, through the narrow gorge pierced by the Irkut through the Tunka Alps, by means of which it now joins the Angara at Irkutsk. At that time, *i.e.* during the post-Tertiary period, its level stood, as shown by the lake deposits and terraces explored by Cherskiy, at least 928 feet above the present level of Lake Baikal, which now lies 1561 feet above the sea level.¹

However, even in its present limits, Lake Baikal occupies the sixth place among the largest lakes of the globe (after Lake Tanganika), and the first place among the Alpine lakes. Sufficient to say that it covers 15,300 square miles, and that the two extremities of the crescent which it makes on a map are 380 miles distant from each other. As for its depth it stands foremost. Already Kononoff's soundings, in 1859, indicated a depth of 5621

feet, and when the Polish exiles, Dr. Dybowski and Godlewski, made, in 1867 and 1871–76, a series of very accurate soundings, they revealed the existence of several valleys in its bottom, attaining depths of 2197, 4460, and 4503 feet, the greatest depths being located in the proximity of the north-western shore, so that a depth of 1935 feet (374 feet below the level of the ocean) was found within a thousand metres from the coast.

Both in its position at the foot of, and the manner it penetrates at its southern extremity into, the plateau, Lake Baikal offers a striking analogy with the Caspian Sea. The same analogy appears in its relations to the surrounding mountains. It is divided about its middle by a submerged ridge, which appears on the surface in the Olkhon Island, and in the promontory of Svyatoi Nos; and of the two basins thus formed, and named respectively the "Great Sea" and the "Small Sea," the southern, that is the one which lies nearest to the plateau, is the deepest. In older works, and in some recent ones as well, Lake Baikal used to be described as a longitudinal valley between two parallel chains of mountains; but it is evident, from what has just been said, how false this view is. The next step would be to consider it as originated from two lakes which once occupied two longitudinal valleys, and joined together after the dividing ridge had been partially destroyed by geological agencies; and this hypothesis, too, has been advocated. Things appear, however, to be much more complicated than that. When I was working out a general scheme of the orography of Siberia, I was compelled to recognise that even the two-valleys-hypothesis could not interpret the real features of the region, and although at that time (1872) we knew next to nothing about the geological structure of the Baikal mountains, I was induced, by considerations about the structure of the plateaus, their border-ridges, and the Alpine chains parallel to the latter, to draw two chains across the northern part of the lake. From the volume under review, we now learn the real state of affairs. In all his explorations in Siberia, Cherskiy used to pay a great deal of attention to the orographical features as they *now* appear to the explorer, and tried to discriminate in how far they were a result of structural features—foldings of the rocks and so on—and in how far they were derived from subsequent erosion which has been going on in these parts of Siberia since the Silurian and Cambrian periods, when the mountain ridges and plateaus received their first shape. As regards the Baikal mountains, it now appears that there is, on the north-western shore, a real ridge running parallel to the shore, and separated by a valley from the mountains lying further west; but that both this ridge and the deep hollow of the Baikal are due, not to structural, but to erosion processes. The ridge consists of slates and gneisses *crossing* it in a diagonal direction, and these strata cross also the northern part of the lake in the same direction—the direction I had indicated on the orographical map on merely theoretical grounds—so as to reappear in the same succession on the eastern shore. The foldings of the Baikal Mountains date from the Silurian, Cambrian, or perhaps even the Laurentian period (Devonian red sandstones lie undisturbed at the outer footings of the Baikal Mountains), but subsequent

¹ There is still a certain uncertainty, perhaps of over 100 feet, concerning the altitude of the level of Lake Baikal. A levelling across Siberia had been made a few years ago; but the death of the person who undertook the calculation of the results brought about some confusion, and Russian geographers suppose that some considerable error may have crept in in the levelling between the Yenisei and Irkutsk, and consequently in the above figure.

erosion and denudation have modified the primary features on a gigantic scale; and a valley so deep as the northern part of Lake Baikal is, has been dug out *across* the former direction of the chains. The lake is thus an immense erosion valley which only partially has been determined by the structural valleys at the foot of the plateau, but has received its final shape through erosion, which made several parallel lakes coalesce as the mountains once separating them were pierced through and obliterated.

This instance will already give an idea of the interest which attaches to the volume now published, and the wealth of data which will be found in it. We sincerely desire, in the interests of geography, that at least these new volumes of the series should be rendered accessible to West European geographers.

The described region is very thinly populated, and contains but few explored remains of the past. As to its flora, it has been properly explored only on the Olkhon Island. The little, however, which is known in these two directions is well summed up, and will give a sound basis for ulterior exploration. We hope to find in the forthcoming volume a summary of all that is known about the fauna of the lake.

P. K.

APPLICATIONS OF BESSEL FUNCTIONS.

A Treatise on Bessel Functions and their Applications to Physics. By Andrew Gray, M.A., and G. B. Mathews, M.A. (London: Macmillan and Co., 1895.)

THIS book, like the kindred work of Prof. Byerly on "Fourier's Series and Spherical Harmonics," marks the modern system of mathematical treatment, and may be contrasted with Dr. Todhunter's "Functions of Laplace, Lamé, and Bessel," of twenty years ago. At that time it was considered desirable to develop the purely mathematical analysis quite apart from the physical considerations to which it owed its life and interest; keeping the pure and the mixed mathematics in separate water-tight compartments, so to speak, with an impenetrable bulkhead between.

But as the Bessel function, like every other function, first presented itself in connection with physical investigations, the authors have done well to begin, on p. 1, with a brief account of three independent problems which lead to its introduction into analysis, before entering upon the discussion of the properties of the Bessel functions.

These three problems are: the small oscillations of a vertical chain, the conduction of heat in a solid cylinder, and the complete solution of Kepler's problem by expressing radius vector, true and excentric anomaly in terms of the mean anomaly.

It is very extraordinary that Kepler's problem should, as a general rule, be still left unfinished in the ordinary treatises, considering that the Bessel function is implicitly defined in the equation; but we need go back only twenty-five years, and we find Boole's "Differential Equations" ignoring the Bessel Function and the solution of the general Riccati equation which it provides. In those days it was customary to speak of any solution, not immediately expressible by algebraical or trigonometrical

functions, as "not integrable in finite terms"; an elliptic integral was skirted round with the remark that it was "reducible to a matter of mere quadrature," and even the homely hyperbolic functions were tabooed.

String is the favourite material of the mathematician for illustrating catenary properties; but it is a relief to find that the authors have provided a *chain* for the discussion of the oscillations when suspended in a vertical line. The banal word *string* turns up accidentally two or three lines lower down (line 10, p. 1), but if a piece of string is used by the side of a length of fine chain, such as is now purchasable, the unsuitability of the string, by reason of its lack of flexibility and its kinkiness, for the representation of catenaries and their oscillations, is at once manifest.

The small plane oscillations of the chain about its mean vertical position are of exactly the same character as the slight deviations from the straight line due to spinning the chain from its highest point of suspension; and this procedure has the advantage of showing a permanent figure, similar to that given for $J_0(\sqrt{x})$ on p. 295 of Lamb's "Hydrodynamics"; with a little practice the knack of producing one, two, three or more nodes at will is easily attained. Thus with a piece of chain 4 feet long, the number of revolutions per second should be 0.54, 1.24, 1.95, 2.65, &c.

The Bessel function was first introduced by the inventor for the complete solution of Kepler's problem, namely, to express the variable quantities in undisturbed planetary motion in terms of the time or mean anomaly $\mu = nt + \epsilon - \omega$.

The authors avoid the awkward integration by parts employed by Todhunter in determining the excentric anomaly ϕ by means of a differentiation. Another procedure will give a/r , where a denotes the mean distance and r the radius vector, more directly, from the relation

$$\phi = \mu + e \sin \phi.$$

For differentiation with respect to μ gives

$$\frac{d\phi}{d\mu} = \frac{1}{1 - e \cos \phi} = \frac{1 + e \cos \theta}{1 - e^2} = \frac{a}{r} = 1 + \Sigma B_r \cos r\mu,$$

suppose, when expressed in a Fourier series, and then

$$B_r = \frac{2}{\pi} \int_0^\pi \cos r\mu \frac{d\phi}{d\mu} d\mu = \frac{2}{\pi} \int_0^\pi \cos r(\phi - e \sin \phi) d\phi = 2J_r(re),$$

according to Bessel's definition.

An integration now gives

$$\phi = \mu + 2 \Sigma \frac{J_r(re)}{r} \sin r\mu$$

and

$$\sin \phi = \frac{\phi - \mu}{e} = 2 \Sigma \frac{J_r(re)}{re} \sin r\mu; \text{ \&c.}$$

Chapters ii.-ix. are devoted to the purely analytical development of the Bessel function, considered as the solution of a differential equation, as an algebraical or trigonometrical series, or as a definite integral; these are the earlier chapters for which the authors apologise in the preface as appearing to contain a needless amount of tedious analysis. In Prof. Byerly's treatise the requisite analysis is introduced in small doses, and only as required; but the ordinary mathematician loves to strew the path at the outside with difficulties best kept out of sight; thus, as Heaviside remarks, the too rigorous mathematician tends to become obstructive. It is of